M1B Schoenbrun Section 6.3 Volumes by Slicing

1) Revolve the function $y = x^2$ around the x axis and find the volume on the interval [0,1]

The slices are circles with radius $r = x^2$ so the area $A(x) = \pi r^2 = \pi\left(x^2\right)^2 = \pi x^4$

so $V = \int_0^1 \pi x^4 dx = \pi\left[\frac{x^5}{5}\right]_0^1 = \pi\left[\frac{1}{5} - 0\right] = \frac{\pi}{5}$

2) Revolve the function $y = x^2$ around the y axis and find the volume on the interval [0,1]

We will integrate on the y axis. The slices are circles with radius $r = x = \sqrt{y}$

so $A(x) = \pi r^2 = \pi\left(\sqrt{y}\right)^2 = \pi y$

so $V = \int_0^1 \pi y\, dy = \pi\left[\frac{y^2}{2}\right]_0^1 = \pi\left[\frac{1}{2} - 0\right] = \frac{\pi}{2}$

3) Revolve the function $y = e^x$ around the x axis and find the volume on the interval [0,1]

The slices are circles with radius $r = e^x$ so the area $A(x) = \pi r^2 = \pi\left(e^x\right)^2 = \pi e^{2x}$

so $V = \int_0^1 \pi e^{2x} dx = \pi\left[\frac{e^{2x}}{2}\right]_0^1 = \pi\left[\frac{e^2}{2} - \frac{1}{2}\right] = \frac{\pi}{2}\left[e^2 - 1\right]$

4) Given a volume with the base a circle of radius 1, with the cross section at each chord perpendicular to a diameter an equilateral triangle, find the volume.

The half chord length $y = \sqrt{1 - x^2}$. This is half the base of the equilateral triangle, so

$A(x) = \frac{1}{2} b \cdot h = \frac{1}{2}\left(2\sqrt{1 - x^2}\right)\cdot\left(2\sqrt{3}\sqrt{1 - x^2}\right) = 2\sqrt{3}\left(1 - x^2\right)$

so
$$v = \int_{-1}^{1} 2\sqrt{3}\left(1 - x^2\right) dx = 2\int_0^1 2\sqrt{3}\left(1 - x^2\right) dx = 4\sqrt{3}\left[x - \frac{x^3}{3}\right]_0^1 =$$
$$4\sqrt{3}\left[\left(1 - \frac{1}{3}\right) - (0 - 0)\right] = 4\sqrt{3}\cdot\frac{2}{3} = \frac{8\sqrt{3}}{3}$$